09 Waves phenomena review answers

Part A: SHM

1. For an oscillation of 8Hz calculate the value of T and ω

   T = 1/f = 0.125s, ω = 2π/T = 50 rad s^{-1}

2. The period of oscillations of a mass of m kg on a spring of stiffness constant k is given by:

   $T = 2\pi\sqrt{(m/k)}$

   a) Calculate the period of oscillation of mass of 4 kg on a spring of stiffness constant 200 Nm^{-1}?

   T = 0.89s (ω = 2π/T = 7.1s^{-1})

   b) If the amplitude of the oscillation is 0.05m calculate the kinetic, potential and total energies of the system when the mass is 0.03m from equilibrium.

   $E_K = \frac{1}{2}m\omega^2(x_0^2 - x^2)$

   $E_{K(max)} = \frac{1}{2}m\omega^2 x_0^2$

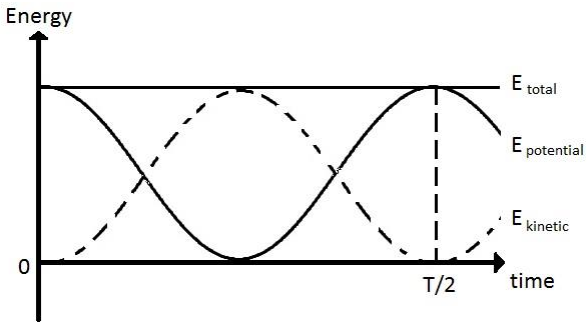
   $E_T = \frac{1}{2}m\omega^2 x_0^2$

   E_k = 0.5x4x7.1^2(0.05^2-0.03^2) = 0.16J

   Total energy = 0.5x4x7.1^{2}x0.05^2=0.25J

   Ep = 0.25 - 0.16 =0.09J

   c) Sketch a graph showing the changing values of kinetic, potential and total over time.

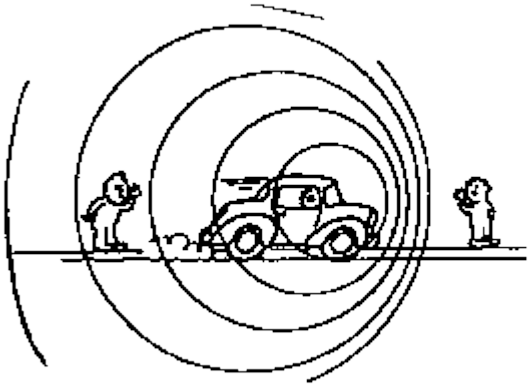


   d) Explain the effect of air resistance and friction in the mass-spring system.

   Air resistance and friction cause damping and so the SHM will gradually reduce in amplitude

Part B: Doppler effect:

1. Two listeners hear the frequency of a moving car engine at differently. The speed of sound is 340ms^{-1} and the speed of the car is 50ms^{-1}. The main frequency of the motor sound is 200Hz.



   a. Explain whether the listener who is in front of the car hear a higher frequency or a lower frequency than the listener behind the car.

   In front of the car the waves are compressed and so more waves arrive per second so frequency is higher.

   b. Calculate the frequency heard by the listener who is behind the car.

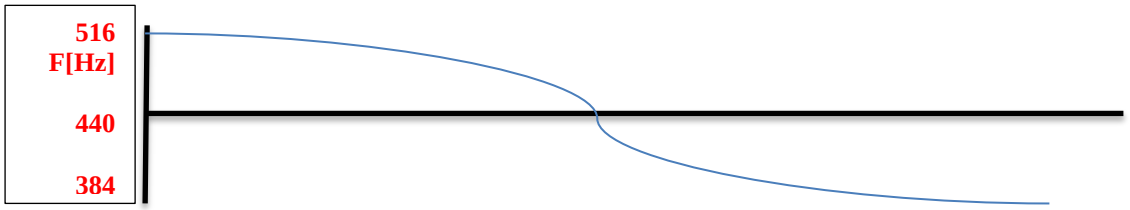
   Moving source:

   F observed = 200(340/(340+50)) = 170 Hz

$$f' = f\left(\frac{v}{v \pm u_s}\right) \quad \text{moving source}$$

$$f' = f\left(\frac{v \pm u_o}{v}\right) \quad \text{moving observer}$$

   c. frequency 440Hz sketch a graph of frequency heard by the driver as the driver passes the listener playing the note. Add values of frequency to the y axis.

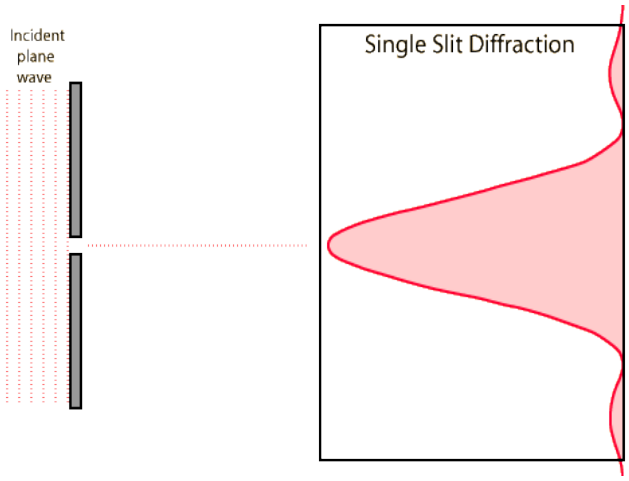


2. A source of 440Hz waves whose speed is 330ms^{-1} waves is travelling at 500ms^{-1}. Calculate the frequencies observed by a stationary listener for this approaching source. What does the negative result mean?
   Moving source formula: f' = f(v-u)/v = 440(330-500)/330 = 440x-170/330 = -230Hz Rounded to 2s.f. the negative value means the observer is receiving the waves in the opposite order that they are sent in.
3. **M31 (the Andromeda galaxy) is approaching us at about 120.0 kms^{-1}. Some light it emits has a wavelength, relative to M31, of 480.0 nm. What is its wavelength as observed byus?**
   Doppler for light formula: Δf = f(v/c)
   f = c/λ = 3x10^{8} ÷ 4.8x10^{-7} = 6.25x10^{14}Hz
   Δf = 6.25x10^{14}(1.2x10^{5}÷3x10^{8}) = 2.5x10^{11}Hz
   So observed frequency is 6.25x10^{14}Hz + 2.5x10^{11}Hz = 6.2525x10^{14}Hz
   Observed wavelength λ = f/c = 3x10^{8}÷6.2525x10^{14}Hz = 479.8 nm
   [Note this answer can be got more easily…….480 x (1-v/c)]

Part C: Diffraction

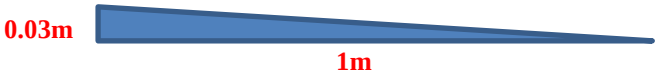
1. When sketching intensity of the single slit diffraction pattern state three key elements of the sketch. **The central maximum is more than 3x higher than other maxima, the intensity is zero at the centre of the minima, the central maximum is significantly wider than the other maxima**

2. The derivation of the formula for finding the angle for the first minimum point in the diffraction pattern comes from assuming the light from each half of the slit will interfere destructively. This occurs when the path difference between the top of the slit and half way down the slit is equal to what value?
   Destructive interference when path difference is half a wavelength.



3. Derive the formula referred to in 2.
   http://nothingnerdy.wikispaces.com/11.3+Diffraction **(derivation is near the end of this web page)**

4. Electromagnetic waves from a source pass through a vertical slit of width 5x10^{-4}m. A detector is used to detect the level of intensity arriving at a distance 1m away from the slit. The detector detects a strong continuous level intensity of radiation along a horizontal line of 5cm length. The detector is kept at the 1m distance. Estimate the wavelength of the radiation being used.
   If radiation is strong for 5cm the distance from centre of maxima to first minima is maybe 3cm.



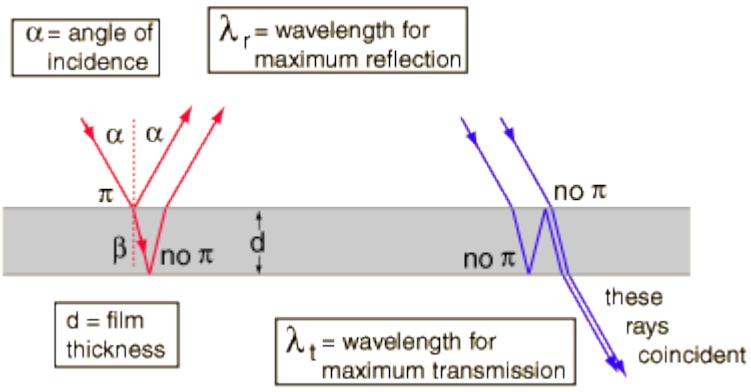
Angle is 0.03 radians.
θ = λ/b
λ = θ b = 0.03 x 5x10^{-4}m= 1.5x10^{-5}m= 0.015 mm

Part D: Interference

1. Some sources of waves are coherent for example a LASER, water waves created by a wave machine, many sound waves. A sufficiently thin slit will make waves coherent. Explain what is meant by a coherent source and why interference patterns are only stable for coherent sources.
   A coherent source is a source where the waves emmited or the waves passing through a specified region are in phase. Interference patterns are only possible when phase difference is constant.
2. Light passes through a diffraction grating which has 16000 slits across a length of 4cm. Calculate the angle for the first order maximum
   $b = 4x10^{-2}/16000 = 2.5x10^{-6}m$
   a. For red light (λ = 660nm)

   Θ = λ/b = 0.66/2.5 = 0.264 radians
   b. For Blue light (λ = 470nm)

   Θ = λ/b = 0.47/2.5 = 0.188 radians
   c. How many red maxima will be visible in total?



There are (π/2)/0.264 in each quadrant = 5
Plus central maximum ➔ 11 maxima

3. A thin film will cause a diffraction pattern due to light travelling through or reflecting off the surfaces. State the path difference that will cause destructive interference due to thin film interference.
   a. For light transmitting though a thin film

   The rays do not undergo phase change so path difference for minima is (n+1/2)λ
   b. For light reflecting off a thin film

   The rays reflecting off the top surface undergoes phase change so path difference for minima is nλ.

   c. A thin oil film floating on water produced thin film interference patterns with a phase change on the first reflection. When light is shining on the film the second maxima for red light is observed. If the refractive index of the oil is 1.46 and red light has a wavelength of 660nm calculate the thickness of the oil film.

Constructive interference: $2dn = \left(m + \frac{1}{2}\right)\lambda$

Destructive interference: $2dn = m\lambda$

The formule needed is the constructive interference one as the is one phase change m = 0 is first constructive interference:
$d = (m+0.5)\lambda/2n = 1.5x660x10^{-9}/2x1.46 = 3.4x10^{-7}m$
Note that this is actually a minimum thickness as the light will have an angle that it is traveling through the film at.

Part E: Resolution

1. When waves pass through a gap or reflect off a narrow surface they diffract.
   a. Explain why smaller wavelengths are needed to accurately detect small objects.
   If the wavelength of light is similar in size to the bumps on a surface the light will diffract and making the bumps undetectable in that wavelength.

b. Explain why an electron of energy 1 MeV can provide more resolution than light.
The de Broglie wavelength of the electron = h/p so it the electron has enough energy its momentum p will be large making its wavelength smaller than light:
$p = mv$ and $E_k = ½ mv^2$. ($E_k = 1x10^6 \times 1.6x10^{-19}$ J)
So $p^2 = 2mE_k = 2 \times 9.11x10^{-31} x1x10^6 \times 1.6x10^{-19}$
$P = 5.399x10^{-22}$ $kgms^{-2}$.
$\lambda = h/p = 6.63x10^{-34} \div 5.399x10^{-22} = 1.2x10^{-12}$m

2. The sand ripples on a beach are approximately 8cm apart. A camera is used to take a picture of a beach from a helicopter flying at a height of 500m. The camera lens has an aperture of 4mm. Explain whether the sand ripples on the beach will be resolved on the image of the camera.
The angle made at the lens by two consecutive ripples is 0.08/500 = $1.6x10^{-4}$ radians.
The resolution angle = 1.22 λ/b = 1.22x4x10-7/$4x10^{-3}$ = $1.22x10^{-4}$ radians so ripples will be just distinguishable in theory.

3. For the diffraction grating in PartD 2. calculate the minimum number of slits that must be illuminated for the red and orange light to be resolvable in the first order spectrum. Take the wavelength of orange light to be 600nm.
Δλ = 660nm-600nm=60nm; m=1
R = λ/Δλ = mN ➔ 660/60 = N = 11 slits.